

例 1.0 (1)

$$(fg)'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta(fg)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{(fg)(x+\Delta x) - (fg)(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x)g(x+\Delta x) - f(x)g(x)}{\Delta x}$$

$$\Delta f = f(x+\Delta x) - f(x) \quad \Delta g = g(x+\Delta x) - g(x)$$

$$\left(\begin{array}{l} \Delta f = f(x+\Delta x) - f(x) \\ \Delta g = g(x+\Delta x) - g(x) \end{array} \right)$$

$$= \lim_{\Delta x \rightarrow 0} \frac{(f(x)+\Delta f)(g(x)+\Delta g) - f(x)g(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{f(x)g(x) + f(x)\Delta g + g(x)\Delta f + \Delta f\Delta g - f(x)g(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \left(f(x) \frac{\Delta g}{\Delta x} + g(x) \frac{\Delta f}{\Delta x} + \frac{\Delta f}{\Delta x} \Delta g \right) = f(x)g'(x) + g(x)f'(x)$$

(1.2.3 とは別々の導き方)

例 2.0

(1) $y = \sqrt{x} = x^{\frac{1}{2}}$ より $y' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

(3) $y' = \frac{(2x^2+3)'(x^2-x) - (2x^2+3)(x^2-x)'}{(x^2-x)^2}$

$$= \frac{4x(x^2-x) - (2x^2+3)(2x-1)}{(x^2-x)^2} = \frac{4x^3 - 4x^2 - (4x^3 - 2x^2 + 6x - 3)}{(x^2-x)^2}$$

$$= \frac{-2x^2 - 6x + 3}{(x^2-x)^2}$$

(4) $y' = \frac{(\sin x)'(1+\cos x) - \sin x(1+\cos x)'}{(1+\cos x)^2}$

例 3.0

(1) $y = \sqrt{2x^2+3}$

$$\begin{cases} u = 2x^2+3 \\ y = \sqrt{u} = u^{\frac{1}{2}} \end{cases}$$

$$y' = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (u^{\frac{1}{2}})' (2x^2+3)' = \frac{1}{2} u^{-\frac{1}{2}} \cdot 4x = \frac{2x}{\sqrt{u}} = \frac{2x}{\sqrt{2x^2+3}}$$

(2) $y = \frac{3}{\sqrt{2x-4}} = 3u^{-\frac{1}{2}} \quad (u = 2x-4)$

$$y' = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (3u^{-\frac{1}{2}})' (2x-4)' = -\frac{3}{2} u^{-\frac{3}{2}} \times 2 = -3u^{-\frac{3}{2}} = -3(2x-4)^{-\frac{3}{2}}$$

(3) $y = \log(e^x+1) = \log u \quad (u = e^x+1)$

$$y' = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (\log u)' (e^x+1)' = \frac{1}{u} e^x = \frac{e^x}{e^x+1}$$

(4) $y = \sin^3 x = u^3 \quad (u = \sin x)$

$$y' = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (u^3)' (\sin x)' = 3u^2 \cos x = 3 \sin^2 x \cos x$$

(5) $y' = (\cos(2x+3))' \cdot 2 = 2 \cos(2x+3)$

(6) $y' = 3(e^{2x})' - 4(e^{5x})' = 3 \cdot 2e^{2x} - 4 \cdot 5e^{5x} = 6e^{2x} - 20e^{5x}$